Digital Image Processing and Pattern Recognition

E1528

Spring 2021-2022

Lecture 6

Lowpass - Smoothing Spatial Filters

INSTRUCTOR

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➢ Contents

➢ Separable Filter Kernels

- ➢ As noted, a 2-D function G(x,y) is said to be separable if it can be written as the product of two 1-D functions, $G_1(x)$ and $G_2(x)$; that is,

$$G(x,y)= G_1(x)\ G_2(x)$$

- ➢ A spatial filter kernel is a matrix, and a separable kernel is a matrix that can be expressed as the outer product of two vectors. For example, the 2*3 kernel

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

➢ Separable Filter Kernels

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- is separable because it can be expressed as the outer product of the vectors

$$c = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } r = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- That is,

$$cr^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = w$$

➢ Separable Filter Kernels

- A separable kernel of size m × n can be expressed as the outer product of two vectors, v and w:

$$w = VW^T$$

Where v and w are vectors of size m × 1 and n × 1, respectively.

- For a square kernel of size m×m , we write

$$w = VV^T$$

- It turns out that the product of a column vector and a row vector is the same as the 2-D convolution of the vectors

Separable Filter Kernels

- The importance of separable kernels lies in the computational advantages that result from the associative property of convolution.

- If we have a kernel w that can be decomposed into two simpler kernels, such that $w=w_{1*}\ w_2$, then it follows from the commutative and associative properties that

$$w * f = (w_1 * w_2) * f = (w_2 * w_1) * f = w_2 * (w_1 * f) = (w_1 * f) * w_2$$

- This equation says that convolving a separable kernel with an image is the same as convolving w_1 with f first, and then convolving the result with w_2.

➢ Separable Filter Kernels

➢ For an image of size M × N and a kernel of size m × n , implementation of Eq. $(w * f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$

requires on the order of **MNmn** multiplications and additions. This is because it follows directly from that equation that each pixel in the output (filtered) image depends on all the coefficients in the filter kernel.

But if the kernel is separable and we use Eq.

$$w * f = (w_1 * w_2) * f = (w_2 * w_1) * f = w_2 * (w_1 * f) = (w_1 * f) * w_2$$

then the first convolution, $w_1 * f$, requires on the order of MNm multiplications and additions because w_1 is of size m × 1.

Separable Filter Kernels

- The result is of size M × N, so the convolution of w_2 with the result requires MNn such operations, for a total of MN(m+n) multiplication and addition operations.
- Thus, the computational advantage of performing convolution with a separable, as opposed to a non-separable, kernel is defined as

$$C = \frac{MN(m+n)}{MNmn} = \frac{m+n}{mn}$$

- For kernels with hundreds of elements, execution times can be reduced by a factor of a hundred or more, which is significant.

Some Important Comparisons Between Filtering in the Spatial and Frequency Domains

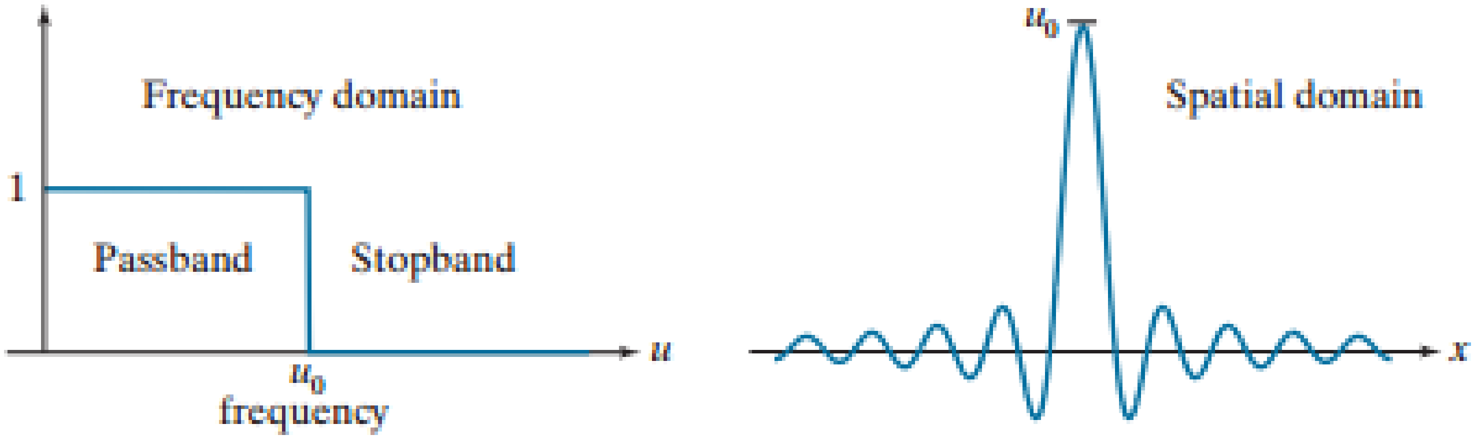
- The tie between spatial and frequency domain processing is the Fourier transform.

- We use the Fourier transform to go from the spatial to the frequency domain; to return to the spatial domain we use the inverse Fourier transform.

a b

(a) Ideal 1-D lowpass filter transfer function in the frequency domain.

(b) Corresponding filter kernel in the spatial domain.



➢ Some Important Comparisons Between Filtering in the Spatial and Frequency Domains

- ➢ The focus here is on two fundamental properties relating the spatial and frequency domains:

1. Convolution, which is the basis for filtering in the spatial domain, is equivalent to multiplication in the frequency domain, and vice versa.

2. An impulse of strength A in the spatial domain is a constant of value A in the frequency domain, and vice versa.

➢ How Spatial Filter Kernels are Constructed

- We consider **Three** basic approaches for constructing spatial filters.
- **1- The first approach** is based on formulating filters based on mathematical properties.
- For example, a filter that computes the average of pixels in a neighborhood blurs an image. Computing an average is similar to integration.
- Conversely, a filter that computes the local derivative of an Image sharpens the image.

How Spatial Filter Kernels are Constructed

- **2- The second approach** is based on sampling a 2-D spatial function whose shape has a desired property.

- For example, we will later show in the samples from a Gaussian function can be used to construct a weighted-average (lowpass) filter.

- These 2-D spatial functions sometimes are generated as the inverse Fourier transform of 2-D filters specified in the frequency domain.

➢ How Spatial Filter Kernels are Constructed

- **3- The third approach** is to design a spatial filter with a specified frequency response.
- This approach is fallen in the area of digital filter design.
- A 1-D spatial filter with the desired response is obtained (typically using filter design software).
- The 1-D filter values can be expressed as a vector v, and a 2-D separable kernel can then be obtained using the equation $w = VV^T$. Or the 1-D filter can be rotated about its center to generate a 2-D kernel that approximates a circularly symmetric function.

Smoothing (Lowpass) Spatial Filters

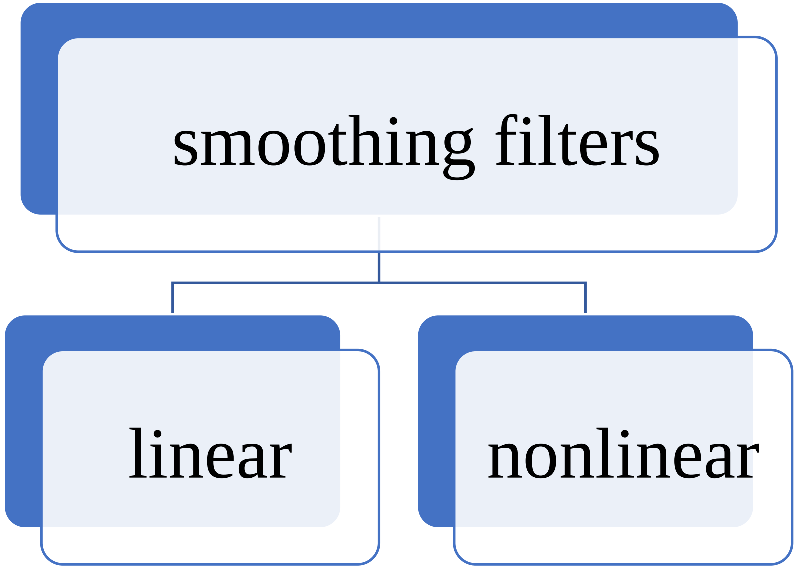
- Smoothing (also called averaging) spatial filters are used to reduce sharp transitions in intensity. Because random noise typically consists of sharp transitions in intensity, an obvious application of smoothing is noise reduction.
- Smoothing prior to image resampling to reduce aliasing, is also a common application.
- Smoothing is used to reduce irrelevant detail in an image, where "irrelevant" refers to pixel regions that are small with respect to the size of the filter kernel.

Smoothing (Lowpass) Spatial Filters

- Smoothing filters are used in combination with other techniques for image enhancement, such as the histogram processing techniques, and unsharp masking, as discussed later.

- We begin the discussion of smoothing filters by considering linear smoothing filters in some detail.

- We will introduce nonlinear smoothing filters later.



Smoothing (Lowpass) Spatial Filters

- As we discussed, linear spatial filtering consists of convolving an image with a filter kernel.

- Convolving a smoothing kernel with an image blurs the image, with the degree of blurring being determined by the size of the kernel and the values of its coefficients.

- In addition to being useful in countless applications of image processing, lowpass filters are fundamental, in the sense that other important filters, including sharpening (high-pass), bandpass, and band-reject filters, can be derived from lowpass filters.

➢ Smoothing (Lowpass) Spatial Filters

- ➢ We discuss in this section lowpass filters based on box and Gaussian kernels, both of which are separable.

- ➢ Most of the discussion will center on Gaussian kernels because of their numerous useful properties and extensiveness of applicability.

➢ Box Filter Kernels

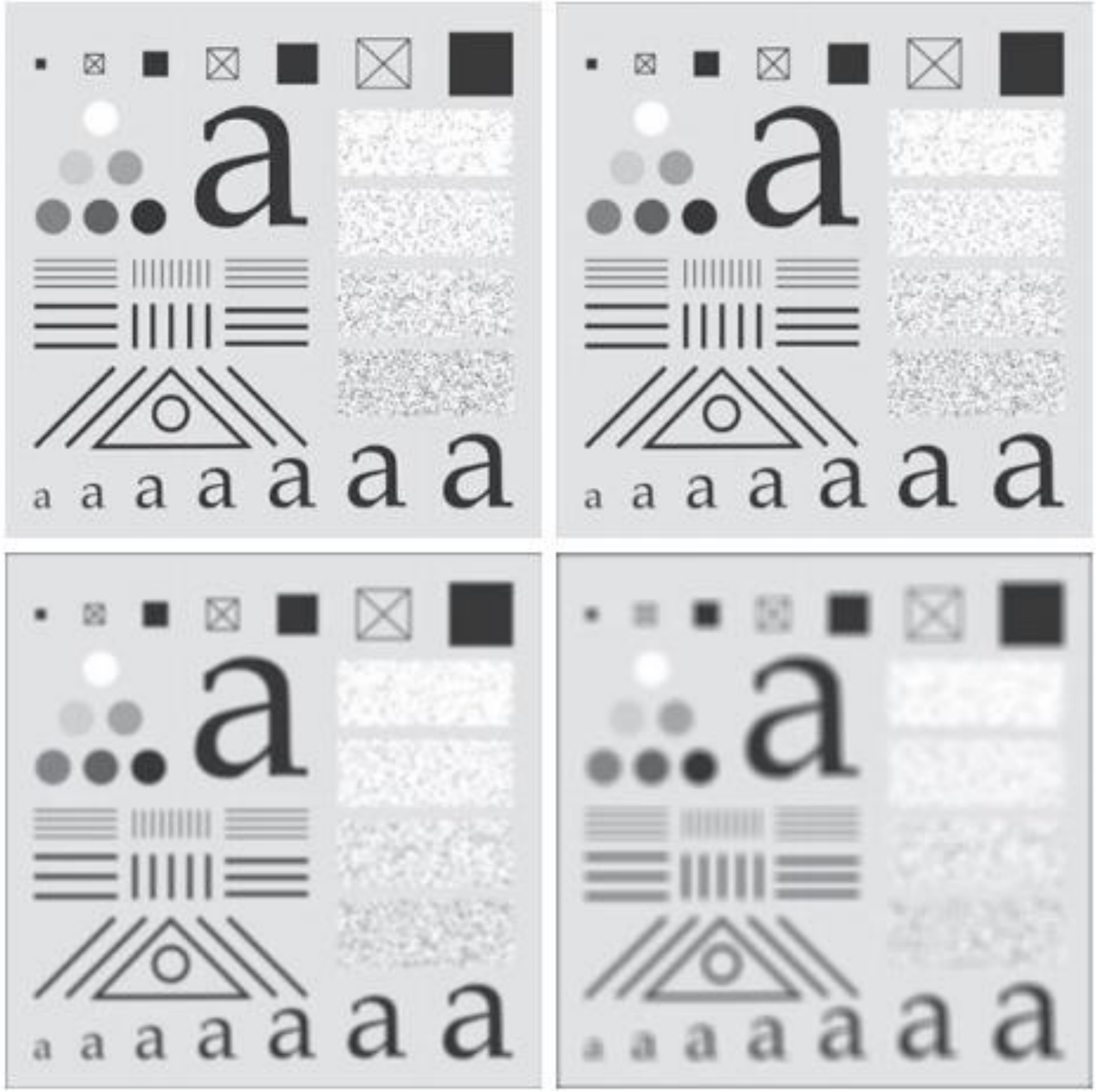
- ➢ The simplest, separable lowpass filter kernel is the box kernel, whose coefficients have the same value (typically 1).
- ➢ The name “box kernel” comes from a constant kernel resembling a box when viewed in 3-D.
- ➢ We showed a 3×3 box filter in next Fig.(a). An m×n box filter is an m×n array of 1’s, with a normalizing constant in front, whose value is 1 divided by the sum of the values of the coefficients (i.e., 1/mn when all the coefficients are 1’s).

➢ Box Filter Kernels

a b
c d

(a) Test pattern of size 1024*1024 · pixels.

(b)-(d) Results of lowpass filtering with box kernels of sizes 3*3, 11*11, and 21*21, respectively.



Box Filter Kernels

- Figure(a) shows a test pattern image of size 1024 × 1024 pixels.
- Figures (b)-(d) are the results obtained using box filters of size m × m with m = 3, 11, and 21 respectively. For m = 3, we note a slight overall blurring of the image, with the image features whose sizes are comparable to the size of the kernel being affected significantly more.
- Such features include the thinner lines in the image and the noise pixels contained in the boxes on the right side of the image. The filtered image also has a thin gray border, the result of zero-padding the image prior to filtering.

Box Filter Kernels

- As indicated earlier, padding extends the boundaries of an image to avoid undefined operations when parts of a kernel lie outside the border of the image during filtering.
- When zero (black) padding is used, the net result of smoothing at or near the border is a dark gray border that arises from including black pixels in the averaging process.
- Using the 11×11 kernel resulted in more pronounced blurring throughout the image, including a more prominent dark border.

Box Filter Kernels

- The result with the 21×21 kernel shows significant blurring of all components of the image, including the loss of the characteristic shape of some components, including, for example, the small square on the top left and the small character on the bottom left.
- The dark border resulting from zero padding is proportionally thicker than before.
- We used zero padding here, and will use it a few more times, so that you can become familiar with its effects.

➢ Lowpass Gaussian Filter Kernels

- ➢ Because of their simplicity, box filters are suitable for quick experimentation, and they often yield smoothing results that are visually acceptable.
- ➢ They are useful also when it is desired to reduce the effect of smoothing on edges.
- ➢ However, box filters have limitations that make them poor choices in many applications. For example, a defocused lens is often modeled as a lowpass filter, but box filters are poor approximations to the blurring characteristics of lenses.

Lowpass Gaussian Filter Kernels

- Another limitation is the fact that box filters favor blurring along perpendicular directions.
- In applications involving images with a high level of detail, or with strong geometrical components, the directionality of box filters often produces undesirable results.
- These are but two applications in which box filters are not suitable.

➢ Lowpass Gaussian Filter Kernels

- The kernels of choice in applications such as those just mentioned are circularly symmetric (also called isotropic, meaning their response is independent of orientation). As it turns out, Gaussian kernels of the form

$$w(s,t) = G(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

are the only circularly symmetric kernels that are also separable.

Thus, because Gaussian kernels of this form are separable, Gaussian filters enjoy the same computational advantages as box filters but have a host of additional properties that make them ideal for image processing.

Lowpass Gaussian Filter Kernels

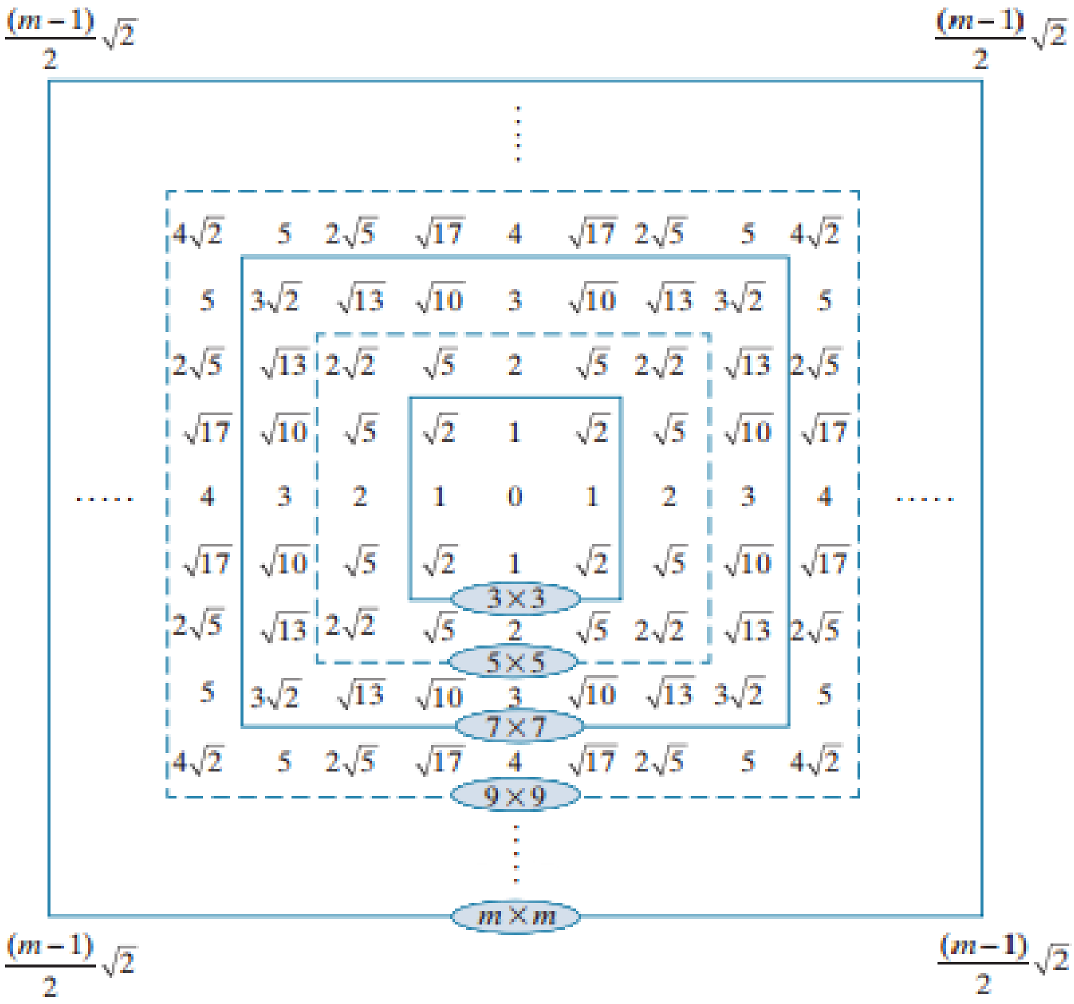
- Variables s and t in last Eq. , are real (typically discrete) numbers.
- By letting $r = \sqrt{s^2 + t^2}$ we can write last equation as

$$G(r) = Ke^{-\frac{r^2}{2\boldsymbol{\sigma}^2}}$$

- This equivalent form simplifies derivation of expressions later in this lecture.
- This form also reminds us that the function is circularly symmetric. Variable r is the distance from the center to any point on function G.

Lowpass Gaussian Filter Kernels

- Figure shows values of r for several kernel sizes using integer values for s and t.
- Because we work generally with odd kernel sizes, the centers of such kernels fall on integer values, and it follows that all values of r^2 are integers also.



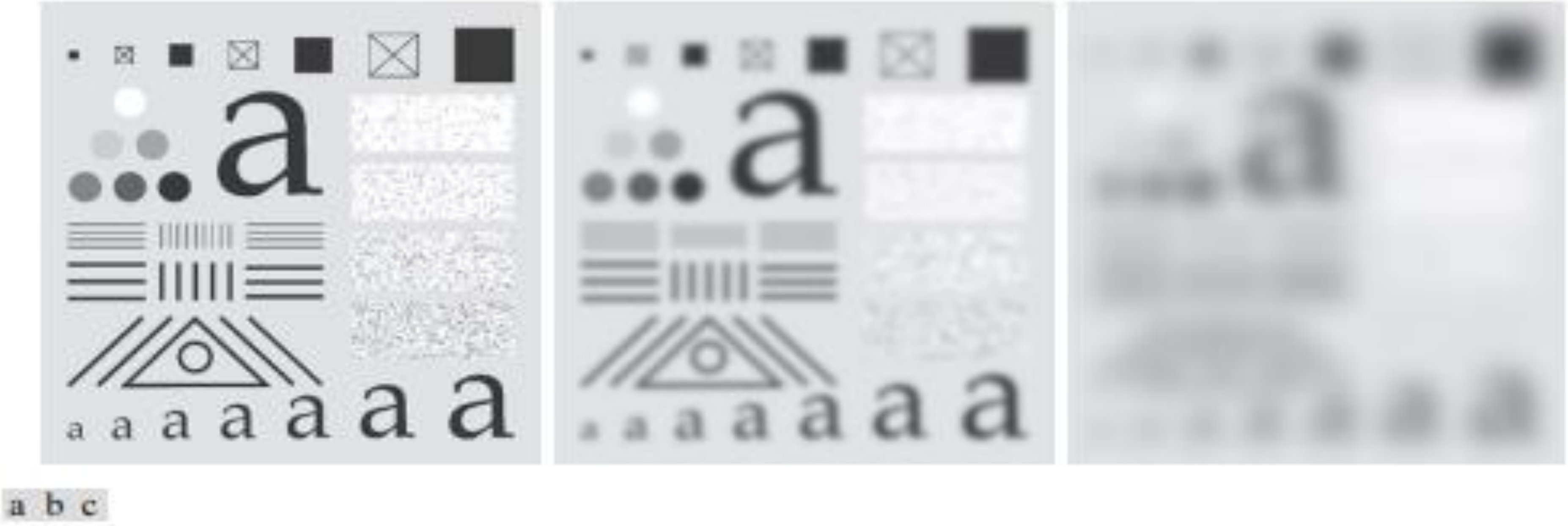
➢ Lowpass Gaussian Filter Kernels

(a)A test pattern of size 1024×1024. (b) Result of lowpass filtering the pattern with a Gaussian kernel of size 21×21, with standard deviations $\sigma = 3.5$. (c) Result of using a kernel of size 43×43 , with $\sigma = 7$. We used K = 1 in all cases.

➢ Lowpass Gaussian Filter Kernels

Result of filtering the test pattern in Fig. using (a) zero padding, (b) mirror padding, and (c) replicate padding. A Gaussian kernel of size 187×187, with $K = 1$ and $\boldsymbol{\sigma} = 31$ was used in all three cases.

➢ Smoothing performance as a function of kernel and image size.



(a) Test pattern of size 4096×4096 pixels. (b) Result of filtering the test pattern with the same Gaussian kernel used in last Fig. (c) Result of filtering the pattern using a Gaussian kernel of size 745×745 elements, with $K = 1$ and $\sigma = 124$. Mirror padding was used throughout.


Thank you